# Optical Microresonators: A Low-Q Introduction Prof. Randall Goldsmith, Dept. of Chemistry, UW Madison

Everything I ever needed to know about microcavities I learned in freshman

chemistry\*

\*not actually true

"Optical microresonators for sensing and transduction: a materials perspective", Advanced Materials, **2017**, 30, 1700037.

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#### **Equations of Motion**

#### LC circuit **Electromagnetic Waves Mechanical Harmonic Oscillator** Wave Equation $\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = 0$ (isotropic, non-W $\vec{E}$ =A(x,y,z)Y(t) polarizable media) $\vec{E} = \sum_{mode\#,s} A(x,y,z)Y(t)$ Helmholtz Eq $\frac{\partial^2}{\partial t^2}Y + \omega^2 Y = 0$ $\frac{\partial^2}{\partial t^2}v + \omega^2 v = 0$ $\frac{\partial^2}{\partial t^2}x + \omega^2 x = 0$ $\nabla^2 \vec{A} + k^2 \vec{A} = 0$ $\omega = kc$ $k = \frac{2\pi}{\lambda}$ $\omega =$ $\omega =$ Spatial modes (later)

Multiple longitudinal modes



Haus, "Waves and Fields in Optoelectronics", 1984, Prentice-Hall

-kx

mg

#### The Quality Factor

$$\frac{\partial^2}{\partial t^2}Y + \omega^2 Y = 0$$

$$Y = Ae^{i\omega t}$$

$$\frac{\partial}{\partial t}Y = i\omega Y$$

 $|Y|^2 \propto energy$ 



But what if your resonator isn't perfect? What if there's loss?

$$\frac{\partial}{\partial t}Y = i\omega Y - \frac{1}{2\tau_0}Y$$
$$Y = Ae^{i\omega t - \frac{1}{2\tau_0}t} = Ae^{i\omega t} e^{-\frac{1}{2\tau_0}t}$$

$$Y = Ae^{i\omega t} \quad {}^{2\tau_0} = Ae^{i\omega t}$$





**Quality Factor** 

$$Q = \tau_0 \omega$$

What could cause Q to drop?

Absorption, scattering, radiative losses, waveguides

Vahala and co-workers, Nature, 2003, 421, 925,

Haus, "Waves and Fields in Optoelectronics", 1984, Prentice-Hall

#### **Coupling to Your Microcavity**

$$\frac{\partial}{\partial t}Y = i\omega Y - \frac{1}{2\tau_0}Y$$
Add an "empty" external w  
partially transmitting mirro

$$\frac{\partial}{\partial t}Y = i\omega Y - \left(\frac{1}{2\tau_0} + \frac{1}{\tau_e}\right)Y$$

vaveguide or r

> Launch a travelling wave at frequency  $\omega_{\rho}$ into that waveguide with power  $|s_{+}|^{2}$

$$\frac{\partial}{\partial t}Y = i\omega Y - \left(\frac{1}{2\tau_0} + \frac{1}{\tau_e}\right)Y + \kappa s_+$$

Use that input wave to "measure" the response of the microcavity as a function of 
$$\omega_e$$

 $s_+ = Be^{i\omega_e t}$ 











You measure the loaded Q, you can potentially calculate the unloaded (intrinsic) Q

Haus, "Waves and Fields in Optoelectronics", 1984, Prentice-Hall; Verhagen, "Microcavities", UvA 2020

**Q** Factor Units!



As 
$$\tau \downarrow$$
;  $\frac{1}{\tau_e} \uparrow$ ;  $\Delta \omega \uparrow$ ;  $Q \downarrow$ 

 $Q = \tau \omega = \frac{2\pi c}{\lambda} \tau$  $Q = \frac{\lambda}{\Delta \lambda} \approx \frac{\omega}{\Delta \omega}$ 



Q=10<sup>8</sup> $\rightarrow$ ~10 fm, ~1 MHz, ~1x10<sup>-5</sup> cm<sup>-1</sup>, 1 neV, etc.



Δω

 $CaF_2$ , Q~10<sup>11</sup>, 5 mm diameter Maleki and co-workers, *PRL*, 102(4), 043902.

#### $\tau \rightarrow$ ~100 ns

Haus, "Waves and Fields in Optoelectronics", 1984, Prentice-Hall; Verhagen, "Microcavities", UvA 2020

ntracavity power

Free Spectral Range (better than Free Range?)



The smaller the L,  $L\downarrow$ ;  $FSR\uparrow$  the larger the FSR



#### That cavity's got Finesse!

Length=L; Refractive index= n





Change in Electric Field (T) 
$$g(\omega) = \sqrt{R_1 R_2} e^{-2i\omega Ln/c}$$
 during one round trip:

After multiple trips:

$$Y_{out} = Y_{in}\sqrt{T_1T_2}e^{-i\omega Ln/c}(1+g(\omega)+g(\omega)^2+\cdots)$$

$$Y_{out} = \frac{Y_{in}\sqrt{T_1T_2}e^{-i\omega Ln/c}}{1-g(\omega)}$$

$$|Y_{out}|^2 = \frac{|Y_{in}|^2 T_1 T_2}{1 + R_1 R_2 - 2\sqrt{R_1 R_2} \cos(\frac{2L\omega n}{c})}$$

$$\Delta \omega_{FWHM} = \frac{\left(1 - \sqrt{R_1 R_2}\right)c}{(R_1 R_2)^{1/4} nL} = \frac{\pi c}{FnL} = \frac{FSR}{F}$$
$$F = \frac{\pi (R_1 R_2)^{1/4}}{1 - \sqrt{R_1 R_2}} \qquad \Delta \omega_{FSR} = \frac{\pi c}{nL} \qquad \text{As F}\uparrow; \Delta \omega \downarrow$$

Haus, "Waves and Fields in Optoelectronics", 1984, Prentice-Hall; Verhagen, "Microcavities", UvA 2020

Q vs F!



As  $F\uparrow;Q\uparrow$ , photons are localized in TIME and FREQUENCY

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Saleh and Teich, "Fundamentals of Photonics", 1991, Wiley; Smith and co-workers, Nanotechnology, 27, 274003, 2016 Haus, "Waves and Fields in Optoelectronics", 1984, Prentice-Hall; Verhagen, "Microcavities", UvA 2020

#### Mode Volume

"...where V is the volume of the resonator"

Physical volume? What about loss?

 $V = \frac{\int dV |E|^2}{\max(|E|^2)}$ 

Normalized. Prone to artifacts (particularly for very low V). Many other definitions. Lumerical even offers multiple options. As  $V \downarrow$ , photons are localized in SPACE Hard to get smaller than  $\lambda^3$  for dielectric microcavity

#### **Microcavity Menagerie**



Fun with Fabry Perot Cavities...



https://en.wikipedia.org/wiki/Optical\_cavity

Planar Fabry Perot Microcavities (are everywhere...)



Planar cavity by Tzu-Ling in my group



Konrad, Meixner and co-workers, Nanoscale 2015, 7, 10204.

Easy to make! Low Q (10-300ish)

No well-defined mode volume, must make use of quasi-normal modes\*

 $V_m = \frac{\pi L^2 \lambda}{1 - R}$ 

Modes along "columns"

Not obviously tunable (ie, L is set)

Can deform cavity Can change k vector!

\*Ujihara, K. Jpn. J. Appl. Phys. 1991, L901.

#### **Planar Fabry Perot Cavities (are sort of tunable)**



https://byjus.com/jee/thin-film-interference/; Dr. Tzu-Ling Chen, Goldsmith group

**Spatial Modes** 

 $\vec{E} = \sum_{mode\#,s} A(x,y,z) Y(t)$ 

confocal

Laguerre-Gaussian modes (cylindrical symmetry) "Transverse Electromagnetic" mode, or  $\text{TEM}_{p\ell}$ 

 $\mathsf{TEM}_{00}$ 

In general, different spatial modes have different energies (Confocal is special, mode spectrum converges)

In an ideal, 1D FP, Transmission can be 100%. But with loss and imperfect mode matching, max % transmission can vary.





Yariv, A. 1991, Optical Electronics, Saunders College Publishing, 4<sup>th</sup> ed.

#### Fabry Perot Microcavities, Part 1



Smith and co-workers, Optics Letters, 35, 2010, 2010 Smith and co-workers, Nanotechnology, 27, 274003, 2016 Developed by Jason Smith @Oxford (other early work from Warburton, Reichel, Sandoghdar)

Generally made by Focused Ion Beam (FIB) milling Then depo\$it high reflectivity optical coating\$



Fig. 2. (Color online) White-light transmission spectra of the cavities: (a) cavity A at  $L = 3.0 \ \mu \text{m}$  and  $L = 12.3 \ \mu \text{m}$ ; (b) HG modes from a single longitudinal mode with (1,0) (0,1) splitting (inset); (c) a high-Q longitudinal resonance (scatter) with\_{15} Lorentzian curve fit (solid curve).

Fabry Perot Microcavities, Part 2, Fiber based!



Developed by David Hunger (now at KIT) with Jakob Reichel@ Sorbonne

- Use a CO<sub>2</sub> laser to smoothly ablate the surface
- Already integrated into photonic infrastructure
- Hunger, Hansch, Reichel, coworkers, NJP 12, 065038 (2010); AIP Adv 2, 02119 (2012)



#### **Fiber Fabry-Perot Microcavities in the Goldsmith Lab**



Vol. 29, No. 2/18 January 2021 / Optics Express 974

#### **Optics EXPRESS**

# Tunable fiber Fabry-Perot cavities with high passive stability

#### CARLOS SAAVEDRA,<sup>1,2,3</sup> DEEPAK PANDEY,<sup>1,4</sup> WOLFGANG ALT,<sup>1</sup> HANNES PFEIFER,<sup>1</sup> AND DIETER MESCHEDE<sup>1</sup>

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2 mn







#### Whispering Gallery Mode Microresonators, Part 1





## How much power?







- 1. WAY too much power
- 2. Too much power
- 3. Not enough power

## Just Right.









#### Whispering Gallery Mode Microresonators, Part 2



, *24*, 20825.

#### Getting to Ultrahigh-Q is not easy



Ever wonder about the **Quality of your Resonators?** How to measure toroidal microresonators using the Nicolas Cage Quality Factor Scale By Kasie Knapper: Friday, January 25th, 10 am, Room 9341 Whispering Gallery Mode



 $\leq Q = \frac{1}{\lambda_{FWHM}}$ 

#### **Spatial Modes**



#### **WGM Notes**

Occasionally referred to as "Travelling Wave" Resonators (no well-defined nodes without defects)

Early modern work on microspheres from Ilchenko (Moscow, JPL)

Coupling via prism or tapered optical fiber

Fiber coupling has many advantages: mode filtering, mode matching, control of coupling (critical coupling @  $\omega = \omega_e$ )

$$\Gamma = \frac{S_{-}}{S_{+}} = \frac{\frac{1}{\tau_{0}} - \frac{1}{\tau_{e}}}{\frac{1}{\tau_{0}} + \frac{1}{\tau_{e}}}$$
Haus, "Waves and Fields in Optoelectronics", 1984, Prer

Microspheres generally "one at a time", vs on-chip toroids

On-chip toroids allow photonic integration (sort of)



Li, Y. L., & Barker, P. F. (2018). Sensors, 18(12), 4184.



Cai, M., Painter, O. and Vahala, K.J., 2000. *Physical review letters*, *85*, 74.

"Jackal"

#### 2D Photonic Crystal Micro(nano)cavities

Imagine a periodic nanostructure with a photonic band gap.  $z_1^{\uparrow}$ 



Add in a defect, microcavity is confined by the surrounding band gap!







Vučković, J., Lončar, M., Mabuchi, H., & Scherer, A. (2001). *Physical Review E*, *65*(1), 016608.

Akahane, Y., Asano, T., Song, B. S., & Noda, S. (2003). *Nature*, *425*(6961), 944-947.

#### 1D Photonic Crystal Micro(nano)cavities; Nanobeams!





Deotare, P. B., McCutcheon, M. W., Frank, I. W., Khan, M., & Lončar, M. (2009). *Applied Physics Letters*, *94*(12), 121106.

Arrays of holes = Bragg mirrors

"zero volume" microcavity

Current frontier: integrating multiple exotic materials (InGaAsP, LiNbO<sub>3</sub>, diamond) into more traditional photonic platforms



L<sub>c</sub> = 1.0 µm <---->



Lee, J., Karnadi, I., Kim, J. T., Lee, Y. H., & Kim, M. K. (2017). ACS Photonics, 4(9), 2117-2123

Burek, M. J., Chu, Y., Liddy, M. S., Patel, P., Rochman, J., Meesala, S., ... & Lončar, M. (2014). *Nature Comm*, *5*(1), 1-7.

#### **Microcavity Menagerie**



#### What is any of this good for? QIS

Photons are great "flying" qubits because they interact weakly with matter

Photons are terrible "stationary" qubits because they interact weakly with matter

How to increase "interaction strength" between photons and quantum systems (atoms, molecules, QDs, defects)?

Solution: Cavity Quantum Electrodynamics (cQED)

How to think about spontaneous emission of a photon (from a molecule)?



ho(E) Photonic density of states (PDOS), all the ways (ie, different k vectors) of coupling to photons



For free space:

 $\rho(\omega) \propto \frac{\omega^2}{\pi c^3}$  Is this actually interesting? Hard to change. As n(refractive index) $\uparrow$ ;  $c\downarrow$ ;  $\rho(\omega)$   $\uparrow$ 

Schatz and Ratner, "Quantum Mechanics in Chemistry," Dover, 2002

#### **The Purcell Effect**

For free space:

$$\rho_{FreeSpace}(\omega) \propto \frac{\omega^2}{\pi c^3}$$

$$\rho_{Cavity}(\omega) \propto \frac{c}{2\pi QV} \frac{3}{\left(\frac{c}{2Q}\right)^2 + (\omega_{cav} - \omega)^2}$$
$$@ \omega \neq \omega_{cav}$$

$$\frac{\Gamma_{Cavity}}{\Gamma_{FreeSpace}} = \frac{\rho_{Cavity}}{\rho_{FreeSpace}} = \frac{3}{16\pi^2 Q} \frac{\lambda^3}{V}$$

Suppressed emission! Can't emit in a band gap!  $\omega = \omega_{cav}$ 



Goy, P., Raimond, J. M., Gross, M., & Haroche, S. (1983). *Physical Review Letters*, *50*, 1903.

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Barnes, W. L., Horsley, S. A., & Vos, W. L. (2020). states. *Journal of Optics*, 22(7), 073501. O. Benson, Quantenoptik09, https://www.physik.hu-berlin.de/de/nano/lehre/copy\_of\_quantenoptik09/Chapter12 Radiative Engineering via the Purcell Effect, Solid State Defects



Reiserer, Merkel, and co-workers, PRX, **2020**, 041025

Evans, Lončar, Lukin, and co-workers, Science, 2018, 362, 662

#### **Radiative Engineering via the Purcell Effect, Molecules**



Wang, Sandoghdar, and co-workers, Nature Photonics, 2019, 15, 483

#### What do you get?

Outcompete relaxation to other (vibronic) states which broadens emission spectrum (photons are more identical)

Faster photon count rates means more information can be transferred, more operations/second

More exotic physics in the "strong coupling" regime



#### **Pro and Cons of Microcavities for QIS**



Good news: tunable resonances!

Bad news: tunable resonances! Instability!

Excellent mode overlap

Used in both large L (ions/Rydberg atoms) and small L (defects) limits

Requires Expen\$ive Optical Coating\$, restricts geometry





WGM

Q up to >10<sup>8</sup> V down to 100's of  $\mu$ m<sup>3</sup>

Not easily tunable

Need to get smooth, limits Q for some materials

Can't make too small, get radiative losses (depends on material, n)

Weak coupling, unless defects are doped inside

Queens of Q's

Fascinating applications in optomechanics



Photonic Crystal/Nanobeam

Q up to ~10<sup>5</sup> V down to <0.6 μm<sup>3</sup>

Not easily tunable

Need to get fabrication defectfree (theoretical Q's  $\sim 10^7$ )

Weak coupling, unless defects are doped inside

Super Low V's (for dielectric)

Fascinating applications in optomechanics



Plasmonic "Nanocavity"

Q up to 10? V down to  $10^{-6} \,\mu\text{m}^3$ 

#### What is any of this good for? Sensing and Spectroscopy



Vollmer and Arnold, Nature Methods., 2008, 5, 591.

K. D. Heylman; K. A. Knapper; E. H. Horak; M. T. Rea;
S. K. Vanga; R. H. Goldsmith. Optical
Microresonators for Sensing and Transduction: A
Materials Perspective. Advanced Materials 2017,
29.

#### Important Benchmarks (not inclusive):

#### Single Particle Detection

Li, Xiao, and coworkers, *PNAS*, **2014**, *111*, 14657. Ozdemir, Yang, and coworkers, *PNAS*, **2014**, *111*, 3836. Lu, Vahala, and coworkers, *PNAS*, **2011**, *108*, 5976. Swaim, Bowen, and coworkers, *APL*, **2013**, *103186*.

Single Protein Detection Dantham, Arnold, and coworkers, *Nano Lett.*, **2013**, *13*, 3347.

Yu and Lu, Nature Comm., 2016, 7, 12311.

Single DNA oligomer, Ion Detection

Baaske, Vollmer, and coworkers, *Nature Nanotech.*, **2014**, *9*, 933.

Baaske, Vollmer, and coworkers, *Nature Photon.*, **2016**, *10*, 733.

# How do you know what molecule has bound?

Surface Functionalization: Bailey, Armani, Hunt, etc.

A means of performing *in situ* spectroscopy would allow single-molecule identification.

### **Optical Microresonator Spectrometers**

Fiber Transmission:







#### **Photothermal Microscopy**



Knapper, Heylman, Horak, and Goldsmith, *Advanced Materials*, 28, 2944, **2016** Heylman, Knapper, and Goldsmith, *Journal of Physical Chemistry Letters*, 5, 1917, **2014** 

#### **Photothermal Spectroscopy**



#### Gold Nanorods:





Heylman, Thakkar, Horak, Quillin, Cherqui, Knapper, Masiello, and Goldsmith, Nature Photonics, 2016, 10, 788



Single Polymer Spectroscopy



#### Watching the Etching of a Single Nanorod via Absorption







Hogan, Horak, Ward, Knapper, Nic Chormaic, and Goldsmith, ACS Nano, 2019, 13, 12743

Hybrid Plasmonic-Photonic Systems



Thakkar, Rea, Smith, Heylman, Quillin, Horak, Knapper, Masiello, and Goldsmith, *Nano Lett.*, **2017**, 17, 6927 Pan, Smith, Nguyen, Knapper, Masiello, and Goldsmith, *Nano Lett.*, **2020**, 20, 50

## Thanks!

Dr. Mike Mattei

Dr. Tzu-Ling Chen

Dr. Lisa-Maria Needham

Professor David Masiello

We are looking for postdocs! Come chat with me!

